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EXPONENTIATED GENERALIZED LINDLEY DISTRIBUTION

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Abstract. The Lindley distribution is important for studying stress–strength reliability modeling. In this paper, we introduce a new model of Lindley distribution referred to as the Exponentiated Generalized Lindley distribution with three parameters which offers a more flexible model for modeling lifetime data. We provide a comprehensive mathematical treatment of this distribution. We derive the expressions for the density function, distribution function, and hazard rate function, moment generating function and r^{th} moment. Distribution of order statistics for the derived distribution is also obtained. We discuss estimation of the parameters by method of maximum likelihood estimation.

Keywords: *exponentiated generalized class distribution, Lindley distribution, moment generating function, Hazard rate function, order statistics, maximum likelihood estimation.*

AMS Subject Classification: *33C90, 62E99, 62E15.*

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1. Introduction

Lifetime distribution represents an attempt to describe, mathematically, the length of the life of a system or a device. Lifetime distributions are most frequently used in the fields like medicine, engineering etc. Many parametric models such as exponential, gamma, Weibull have been frequently used in statistical literature to analyze lifetime data. Recently, one parameter Lindley distribution has attracted the researchers for its use in modeling lifetime data. Because of having only one parameter, the Lindley distribution does not provide enough flexibility for analyzing different types of lifetime data. To increase the flexibility for modeling purposes it will be useful to consider further generalizations of this distribution. This paper offers a three-parameter family of distributions which generalizes the Lindley distribution.

The continuous one parameter Lindley distribution was introduced by Lindley (1958) [9]. Lindley used the distribution named after him to illustrate a difference between fiducially distribution and posterior distribution. The probability density function (pdf) of Lindley distribution is given by:

$$g(x) = \frac{\theta^2}{\theta+1}(1+x)\cdot e^{-\theta.x}, x > 0,\ \theta > 0$$

is a two-component mixture of an exponential distribution with scale parameter θ and gamma distribution with shape parameter 2 and scale parameter θ. The mixing proportion is p=θ/(θ+1). Many generalizations of the Lindley distribution have been proposed in recent years. The generalizations that we are aware of: Sankaran (1970) [13] derived the Poisson-Lindley distribution. The Poisson-Lindley distribution provided a better fit to the empirical set of data considered than the negative binomial and Hermite distributions. Ghitany et al. (2008, 2011) [6, 7] studied various properties of Lindley distribution and derived the two-parameter weighted Lindley distribution with applications. He pointed that Lindley distribution is particularly useful in modeling biological data from mortality studies. Zakerzadeh and Dolati (2009) [15] have proposed a new two parameter lifetime distribution, as generalized Lindley (GL) distribution. Bakouch et al. (2012) [2] introduced an extension of the Lindley distribution that offers more flexibility in the modeling of lifetime data. Ghitany et al. (2013) [5] presented results on the two-parameter generalization referred to as the power Lindley distribution. The Weibull Lindley (WL) distribution due to Asgharzadeh et al. (2014a) [1]. Warahena-Liyanage and Pararai (2014) [14] introduced the generalized power Lindley (GPL) model which is a more flexible model for the lifetime data than the power Lindley model. Merovci (2014) [10] introduced the Beta-Lindley Distribution: Properties and Applications. Oluyede and Yang (2014) [12] introduced the beta generalized Lindley distribution. Elbatal et al. (2015) [4] introduced a new class of Generalized Power Lindley Distribution. Oluyede, Yang and Omolo (2015) [11] derived the generalized class of Exponential Kumarswamy Lindley distribution.

The paper is organized as follows. In Section 2 & 3, we define Lindley distribution and Exponentiated Generalized class distribution. In Section 4 we provide the probability density function (pdf) and the cumulative distribution function (cdf) of the Exponentiated Generalized Lindley distribution. In Section 5 we discuss the hazard rate function and survival function for the new distribution. Formulas for moments and moment generating function of the Exponentiated Generalized Lindley distribution are given in Section 5 & 6. The distribution of the order statistics for the new distribution is discussed in Section 7. We use the method of maximum likelihood estimation to estimate its parameters in Section 8. The following Lemmas will also be needed to complete the derivations:

Lemma 1. From Gradshteyn and Ryzhik (2007) [8], Equation (3.381.4), Page 346.
For $Re\,\nu > 0, Re\,\mu > 0$,

$$\int_0^\infty x^{\nu-1} e^{-\mu x} dx = \frac{\Gamma \nu}{\mu^{\nu}}.$$

Lemma 2. From Gradshteyn and Ryzhik (2007) [8], Equation (1.110), Page 25.
If α is a positive real non integer and $|x| \leq 1$, then by binomial series expansion we have:

$$(1-x)^{\alpha-1} = \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} x^j.$$

2. Lindley Distribution

A continuous random variable X is said to have a Lindley distribution, if its pdf $g(x)$ and $cdf\ G(x)$ are, respectively, given by:

$$g(x) = \frac{\theta^2}{\theta+1}(1+x).e^{-\theta.x}, \quad x>0, \theta>0, \tag{1}$$

and

$$G(x) = 1 - \frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}. \tag{2}$$

3. Exponentiated Generalized Class of Distributions

Cordeiro et al. (2013) proposed a new class of distributions that extend the exponentiated type distributions and they obtained some of its structural properties. Given a continuous cdf $G(x)$, they defined the *cdf* of the Exponentiated Generalized (EG) class of distributions by

$$F(x) = \left[1 - \{1 - G(x)\}^{\alpha}\right]^{\beta}, \tag{3}$$

where $\alpha > 0$ and $\beta > 0$ are two additional shape parameters. The probability density function (pdf) of the new class has the form

$$f(x) = \alpha\beta\{1 - G(x)\}^{\alpha-1}\left[1 - \{1 - G(x)\}^{\alpha}\right]^{\beta-1} g(x). \tag{4}$$

The Exponentiated Generalized (EG) family of densities (4) allows for greater flexibilityof its tails and can be widely applied in many areas of engineering and biology.

4. Derivation of CDF and PDF

In this section, we introduce the three-parameter Exponentiated Generalized Lindley (EGL) distribution. Using (1) & (2) in (4), the pdf of the (EGL) distribution can be written as

$$f(x) = \alpha\beta\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha-1}\left[1 - \left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta-1} . \frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}.$$

Using binomial expansion in Lemma 2, we get

$$\begin{aligned} f(x) &= \frac{\alpha\beta\theta^2}{\theta+1}(1+x)e^{-\theta x}\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha-1}\sum_{j=0}^{\infty}\binom{\beta-1}{j}(-1)^j\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha j} \\ &= \frac{\alpha\beta\theta^2}{\theta+1}\sum_{j=0}^{\infty}\binom{\beta-1}{j}(-1)^j(1+x).\left(1+\frac{\theta}{\theta+1}x\right)^{\alpha j+\alpha-1} e^{-\theta\alpha(1+j).x} \\ &= \frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^j\binom{\beta-1}{j}.\binom{\alpha+\alpha j-1}{k}(1+x)\cdot x^k e^{-\theta\alpha(1+j).x}. \end{aligned} \tag{5}$$

From (3) & (2), cdf of the new distribution can be defined as:

$$F(x) = \left[1 - \left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta}. \tag{6}$$

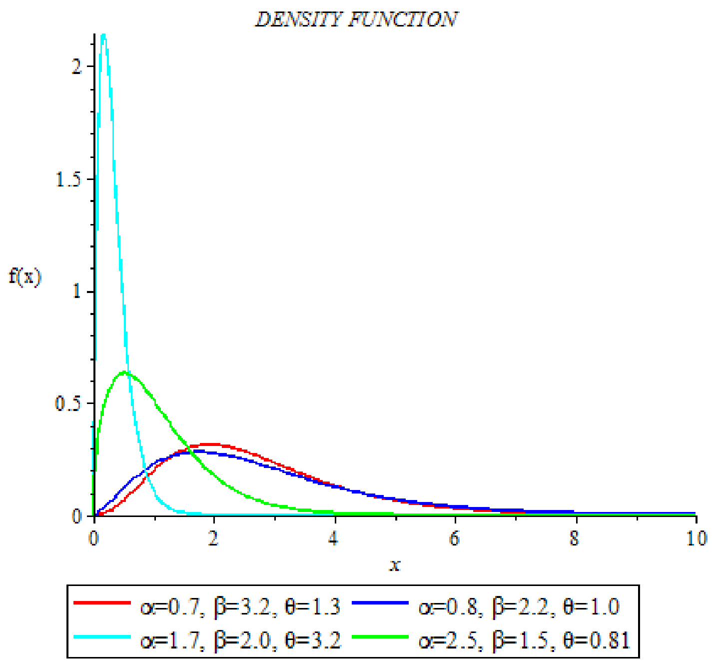

Fig. 1. Graph of pdf of EGL distribution for different values of its parameters α, β and θ

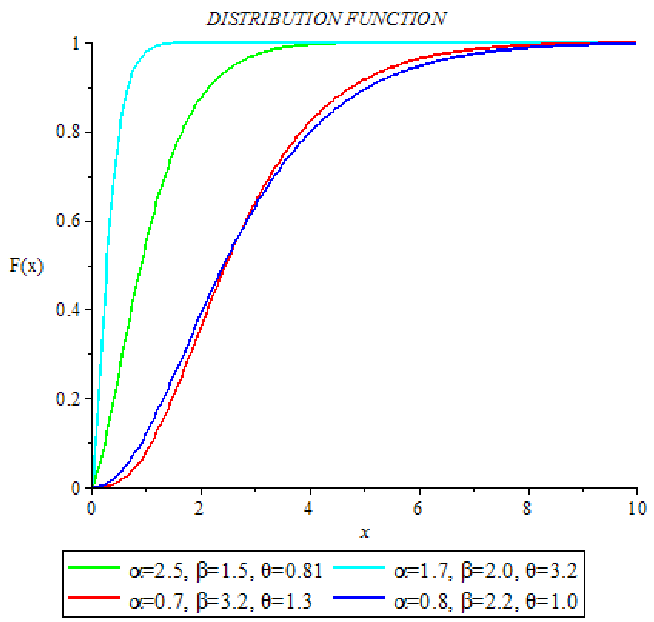

Fig. 2. Graph of cdf of EGL distribution for different values of its parameters α, β and θ

5. Hazard Rate Function And Survival Function

The hazard rate function defined by $h(x)=\dfrac{f(x)}{1-F(x)}$ is an important quantity characterizing life phenomena. For the pdf defined in equation (5), $h(x)$ takes the form:

$$h(x)=\frac{\alpha\beta\left(\dfrac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha-1}\left[1-\left(\dfrac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta-1}.\dfrac{\theta^2}{\theta+1}(1+x)e^{-\theta x}}{1-\left[1-\left(\dfrac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta}} \quad (7)$$

and its survival function is given by:

$$S(x)=1-F(x),$$

$$S(x)=1-\left[1-\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta}. \quad (8)$$

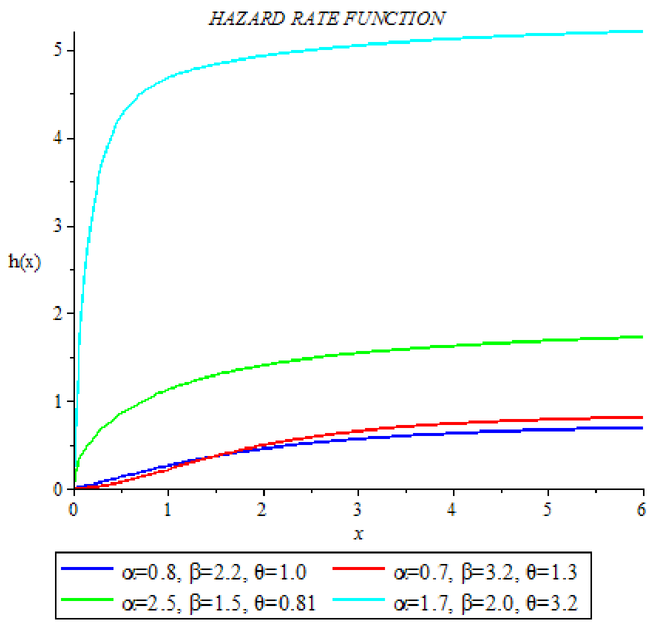

Fig.3. Graph of hazard rate function of EGL distribution for different values of its parameters α, β and θ.

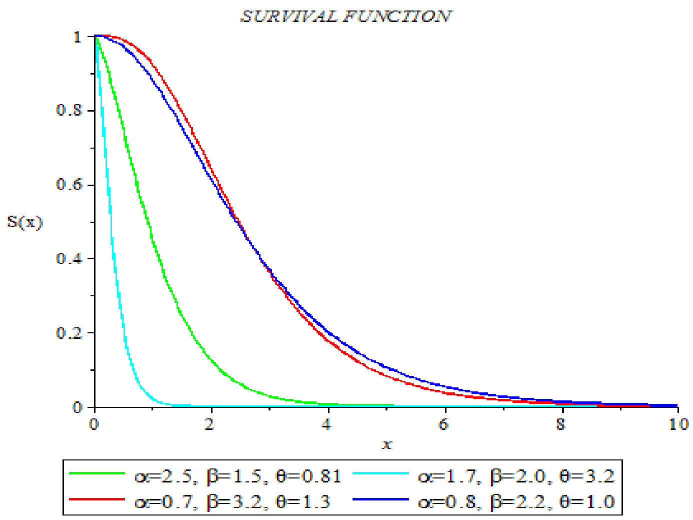

Fig 4. Graph of survival function of EGL distribution for different values of its parameters α, β and θ

6. Moments

If a random variable X has the pdf given by equation (5), then the corresponding $r-th$ moment is given by:

$$\mu_r^{'} = E\left(x^r\right) =$$

$$= \frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^j \binom{\beta-1}{j} . \binom{\alpha+\alpha j-1}{k} \int_0^{\infty} x^{k+r} . e^{-\theta\alpha(1+j).x} dx +$$

$$\frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^j \binom{\beta-1}{j} . \binom{\alpha+\alpha j-1}{k} \int_0^{\infty} x^{k+r+1} . e^{-\theta\alpha(1+j).x} dx.$$

Using Lemma 1, we get

$$\mu_r^{'} = \frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^j \binom{\beta-1}{j} . \binom{\alpha+\alpha j-1}{k} \frac{\Gamma(k+r+1)}{\{\theta\alpha(j+1)\}^{(k+r+1)}} + \frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^j \binom{\beta-1}{j} . \binom{\alpha+\alpha j-1}{k} \frac{\Gamma(k+r+2)}{\{\theta\alpha(j+1)\}^{(k+r+2)}}. \quad (9)$$

7. Moments Generating Function

The different moments of proposed distribution can also be obtained by using the moment generating function (mgf). If the random variable X has the density function (5), then the moment generating function $M_X(t)$, of X is given by:

$$M_X(t) = E(e^{tX}) =$$

$$=\frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{j}\binom{\beta-1}{j}.\binom{\alpha+\alpha j-1}{k}\int_{0}^{\infty}x^{k}.e^{-(\theta\alpha(1+j)-t).x}dx$$

$$\frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{j}\binom{\beta-1}{j}.\binom{\alpha+\alpha j-1}{k}\int_{0}^{\infty}x^{k+1}.e^{-(\theta\alpha(1+j)-t).x}dx.$$

Using Lemma 1, we get

$$M_X(t)=\frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{j}\binom{\beta-1}{j}.\binom{\alpha+\alpha j-1}{k}\frac{\Gamma(k+1)}{\{\theta\alpha(j+1)-t\}^{(k+1)}}$$
$$\frac{\alpha\beta\theta^{k+2}}{(\theta+1)^{k+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{j}\binom{\beta-1}{j}.\binom{\alpha+\alpha j-1}{k}\frac{\Gamma(k+2)}{\{\theta\alpha(j+1)-t\}^{(k+2)}}. \tag{10}$$

8. Order Statistics

In this section, we derive closed form expressions for the pdf*s* of the $i-th$ order statistic of the Exponentiated Generalized Lindley (EGL) distribution. Let $X_1, X_2, ..., X_n$ be a simple random sample from (EGL) distribution with pdf and cdf given by (5) and (6), respectively. Let $X_{(1:n)} \le X_{(2:n)} \le ... \le X_{(n:n)}$ denote the order statistics obtained from this sample. We now give the probability density function of $X_{r:n}$, say $f_{r:n}(x)$ and the moments of $X_{r:n}$, $i=1,2,...,n$. The probability density function of the $r-th$ order statistics $X_{r:n}$, $r=1,2,...,n$ given by (see, David (1981) [3])

$$f_{r:n}(x)=C_{r:n}[F(x;\phi)]^{r-1}[1-F(x;\phi)]^{n-r}f(x;\phi),\ x>0, \tag{11}$$

where $F(.)$ and $f(.)$ are given by (3) and (4) respectively, and

$$C_{r:n}=\frac{n!}{(r-1)!(n-r)!}.$$

Thus,

$$f_{r:n}(x)=C_{r:n}.\alpha\beta\{1-G(x)\}^{\alpha-1}\left[1-\{1-G(x)\}^{\alpha}\right]^{\beta.r-1}\left[1-\left\{1-\{1-G(x)\}^{\alpha}\right\}^{\beta}\right]^{n-r}g(x),$$

$$f_{r:n}(x)=C_{r:n}.\alpha\beta\sum_{k=0}^{n-r}(-1)^{k}\binom{n-r}{k}\{1-G(x)\}^{\alpha-1}\left[1-\{1-G(x)\}^{\alpha}\right]^{\beta.(r+k)-1}g(x).$$

Using binomial expansion given in Lemma 2, we get

$$f_{r:n}(x)=C_{r:n}.\alpha\beta\sum_{k=0}^{n-r}\sum_{j=0}^{\infty}(-1)^{k+j}\binom{n-r}{k}.\binom{\beta.(r+k)-1}{j}\{1-G(x)\}^{\alpha(j+1)-1}g(x),$$

$$f_{r:n}(x)=C_{r:n}.\alpha\beta\sum_{k=0}^{n-r}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{k+j+m}\binom{n-r}{k}.\binom{\beta.(r+k)-1}{j}.\binom{\alpha(j+1)-1}{m}.\{G(x)\}^{m}g(x),$$

$$f_{r:n}(x)=C_{r:n}.\alpha\beta\sum_{k=0}^{n-r}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{k+j+m}\binom{n-r}{k}.\binom{\beta.(r+k)-1}{j}.\binom{\alpha(j+1)-1}{m}$$
$$\times\left\{1-\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right\}^{m}.\frac{\theta^{2}}{\theta+1}(1+x).e^{-\theta.x},$$
$$f_{r:n}(x)=C_{r:n}\cdot\frac{\alpha\beta\theta^{2}}{\theta+1}w_{kjmi}\left\{1+\frac{\theta}{\theta+1}x\right\}^{i}\cdot(1+x)\cdot e^{-\theta.x(i+1)}, \tag{12}$$

where,
$$w_{kjmi}=\sum_{k=0}^{n-r}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\sum_{i=0}^{\infty}(-1)^{k+i+j+m}\binom{n-r}{k}\cdot\binom{\beta.(r+k)-1}{j}\cdot\binom{\alpha(j+1)-1}{m}\cdot\binom{m}{i}.$$

Thus, the p^{th} moment for the r^{th} order statistics of proposed distribution is given by:
$$\mu_{r:n}^{(p)}(x)=C_{r:n}\cdot\frac{\alpha\beta\theta^{2}}{\theta+1}w_{kjmi}\int_{0}^{\infty}x^{p}\left\{1+\frac{\theta}{\theta+1}x\right\}^{i}\cdot(1+x).e^{-\theta.x(i+1)}dx.$$

Using binomial expansion given in Lemma 2, we get
$$\mu_{r:n}^{(p)}(x)=C_{r:n}\cdot\frac{\alpha\beta\theta^{2}}{\theta+1}w_{kjmi}\sum_{q=0}^{\infty}\binom{i}{q}\int_{0}^{\infty}x^{p}\left\{\frac{\theta}{\theta+1}x\right\}^{q}\cdot(1+x)\cdot e^{-\theta.x(i+1)}dx,$$
$$\mu_{r:n}^{(p)}(x)=C_{r:n}\cdot\frac{\alpha\beta\theta^{2}}{\theta+1}w_{kjmiq}\int_{0}^{\infty}x^{p}\left\{\frac{\theta}{\theta+1}x\right\}^{q}\cdot(1+x)\cdot e^{-\theta.x(i+1)}dx,$$
$$w_{kjmiq}=\sum_{k=0}^{n-r}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\sum_{q=0}^{\infty}(-1)^{k+i+j+m}\binom{n-r}{k}.\binom{\beta.(r+k)-1}{j}.\binom{\alpha(j+1)-1}{m}.\binom{m}{i}.\binom{i}{q},$$

$$\mu_{r:n}^{(p)}(x)=C_{r:n}.w_{kjmiq}\frac{\alpha\beta\theta^{2+q}}{(\theta+1)^{q+1}}\int_{0}^{\infty}x^{p+q}.e^{-\theta.x(i+1)}dx+C_{r:n}.w_{kjmiq}\frac{\alpha\beta\theta^{2+q}}{(\theta+1)^{q+1}}\int_{0}^{\infty}x^{p+q+1}.e^{-\theta.x(i+1)}dx.$$

Using Lemma 1, we get
$$\mu_{r:n}^{(p)}(x)=C_{r:n}.w_{kjmiq}\frac{\alpha\beta\theta^{2+q}}{(\theta+1)^{q+1}}\frac{\Gamma(p+q+1)}{\{\theta(i+1)\}^{(p+q+1)}}+C_{r:n}.w_{kjmiq}\frac{\alpha\beta\theta^{2+q}}{(\theta+1)^{q+1}}\frac{\Gamma(p+q+2)}{\{\theta(i+1)\}^{(p+q+2)}}. \tag{13}$$

9. Maximum Likelihood Estimators

Let X be a random variable having the pdf of EGL distribution defined as:
$$f(x)=\alpha\beta\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha-1}\left[1-\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta-1}.\frac{\theta^{2}}{\theta+1}(1+x)e^{-\theta x},$$
$$f(x)=\alpha\beta\left(1+\frac{\theta}{\theta+1}x\right)^{\alpha-1}\left[1-\left(\frac{\theta+1+\theta.x}{\theta+1}e^{-\theta.x}\right)^{\alpha}\right]^{\beta-1}.\frac{\theta^{2}}{\theta+1}(1+x)e^{-\theta x\alpha}.$$

Then its log-likelihood function can be written as:

$$L(x;\alpha,\beta,\theta) = n\ln\alpha + n\ln\beta + 2n\ln\theta - n\ln(\theta+1) + \sum_{i=0}^{\infty}\ln(1+x_i) + (\alpha-1)\sum_{i=0}^{\infty}\ln\left(1+\frac{\theta}{\theta+1}x_i\right) - \theta\alpha\sum_{i=0}^{\infty}x_i + (\beta-1)\sum_{i=0}^{\infty}\ln\left(1-\left(\frac{\theta+1+\theta.x_i}{\theta+1}e^{-\theta.x_i}\right)^{\alpha}\right). \tag{14}$$

Thus the non-linear normal equation is given as follows:

$$\frac{\partial L(x;\alpha,\beta,\theta)}{\partial\beta} = \frac{n}{\beta} + \sum_{i=0}^{\infty}\ln\left(1-\left(\frac{\theta+1+\theta.x_i}{\theta+1}e^{-\theta.x_i}\right)^{\alpha}\right), \tag{15}$$

$$\frac{\partial L(x;\alpha,\beta,\theta)}{\partial\theta} = \frac{2n}{\theta} - \frac{n}{\theta+1} - \alpha\sum_{i=0}^{\infty}x_i + (\alpha-1)\sum_{i=0}^{\infty}\frac{x_i/(\theta+1)^2}{\left(1+\frac{\theta}{\theta+1}x_i\right)} (\beta-1)\sum_{i=0}^{\infty}\frac{\alpha e^{-\theta\alpha.x_i}\left(1+\frac{\theta}{\theta+1}x_i\right)^{\alpha-1}\left\{x_i\left(1+\frac{\theta}{\theta+1}x_i\right)-\frac{1}{(\theta+1)^2}\right\}}{\left(1-\left(\frac{\theta+1+\theta.x_i}{\theta+1}e^{-\theta.x_i}\right)^{\alpha}\right)}. \tag{16}$$

$$\frac{\partial L(x;\alpha,\beta,\theta)}{\partial\alpha} = \frac{n}{\alpha} - \theta\sum_{i=0}^{\infty}x_i + \sum_{i=0}^{\infty}\ln\left(1+\frac{\theta}{\theta+1}x_i\right) (\beta-1)\sum_{i=0}^{\infty}\frac{e^{-\theta\alpha.x_i}(\theta.x_i)\left(1+\frac{\theta}{\theta+1}x_i\right)^{\alpha}\ln\left(1+\frac{\theta}{\theta+1}x_i\right)}{\left(1-\left(\frac{\theta+1+\theta.x_i}{\theta+1}e^{-\theta.x_i}\right)^{\alpha}\right)}. \tag{17}$$

We can find the estimates of the unknown parameters by maximum likelihood method by setting these above non-linear equations (15)-(17) to zero and solve them simultaneously.

10. Conclusion

In this paper the expressions for the cdf and pdf of Exponentiated Generalized Lindley (*EGL*) distribution are derived in section 4. The effect of parameters is evident from graphs drawn for derived pdf and cdf for different combination of parameters. In general, graphs drawn for Exponentiated Generalized Lindley distribution depicts that distribution is unimodal and positively skewed. The hazard rate function, survival function and their graphs for new distribution are given in section 5. The expressions for its $r-th$ moment and mgf are of derived distribution are given in equation (9) and equation (10) respectively. The expressions for its pdf of the $r-th$ order statistics and moments are derived in equation (12) and equation (13) respectively. The method of MLE to estimate its parameters is discussed in Section 9.

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